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JOINT ONE-DIMENSIONAL INVERSE DYNAMIC PROBLEMS FOR SYSTEMS OF HYPERBOLIC EQUATIONS

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KEYWORDS

pore-elasticity, exact problem, inverse problem, wave equation, dynamic problem, NE wave, continuity, correctness, Darboux problem, Volterra integral equation, canonical form, uniqueness of solution , stagnation

ABSTRACT

Joint one-dimensional inverse dynamic problems in the porous-elastic medium are considered: for the two-dimensional porous-elastic equation representing the propagation process of the NE wave in the porous half-space, the momentum acting only on the depth and at the boundary of the half-space the problem of determining one of the four parameters of the medium structure independent of the unknown shape of the point source is considered. It has been proved that under certain assumptions about the structure of the source and environment, both one-dimensional unknown functions are single-valued given the displacement of the boundary points. The stability estimates of the solution of the problem are presented.

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GIPERBOLIK TENGLAMALAR SISTEMASI UCHUN BIRGALIKDAGI BIR O‘LCHAMLI TESKARI DINAMIK MASALALAR

KALIT SO‘ZLAR/ КЛЮЧЕВЫЕ СЛОВА:
g’ovak-elastiklik, to’g’ri masala, teskari masala, to’lqin tenlamasi, dinamik masala, SH to’lqin, uzluksizlik, korrektlik, Darbu masalasi, Volterra integral tenglamasi, kanonik ko‘rinish, yechim yagonaligi, turg’unlik

ANNOTATSIYA/ АННОТАЦИЯ

G‘ovak-elastik muhitda birgalikdagi bir o‘lchamli teskari dinamik masalalar qaralgan: g‘ovak yarim fazoda SH to‘lqinni tarqalish jarayonini ifodalaydigan ikki o‘lchamli g‘ovak-elastik tenglama uchun faqat chuqurligiga va yarim fazoning chegarasida ta’sir etuvchi impuls nuqtali manbaning noma’lum shakliga bog‘liq bo‘lmagan muhit strukturasining to‘rtta parametridan bittasini aniqlash haqidagi masala qaralgan. Isbotlanganki, manba va muhit strukturasining tuzilishi haqidagi ma’lum bir farazlarda ikkala bir o‘lchovli noma’lum funksiyaning ikkalasi ham chegara nuqtalarining siljishi berilishi bilan bir qiymatli aniqlanadi. Masala yechimining turg‘unlik baholari keltirilgan.

MASALANING QO‘YILISHI

$G = R_+^2 \times R$, $R_+^2 = \{(x,y) \in R^2 \mid y > 0\}$ yarim fazoda $u(x,y,t)$, $v(x,y,t)$ funksiyalarga nisbatan

$$\rho_s u_{tt} = (\mu u_x)_x + (\mu u_y)_y - b\rho_l (u_t - v_t),\ (x,y,t) \in G, \tag{1}$$

$$\rho_l v_t = b\rho_l (u - v),\ (x,y,t) \in G \tag{2}$$

differensial tenglamalar sistemasini qaraylik. $\mu = \mu(y), \rho_s = \rho_s(y)$ koeffitsentlari $C^2(R_+)$ sinfda musbat funksiya, $b = \chi\rho_l, \chi = \chi(y)$, $\rho_l = \rho_l(y)$ esa $C^1(R_+)$ sinfda musbat funksiyalar. $R_+ = \{x \in R \mid x > 0\}$.

Aytaylik, $u(x,y,t)$, $v(x,y,t)$ funksiyalar (1),(2) tenglamadan tashqari quyidagi boshlang‘ich chegaraviy shartlarni qanoatlantirsin:

$$u|_{t<0} \equiv 0,\ v|_{t<0} \equiv 0. \tag{3}$$

$$\mu u_y|_{y=0} = f(t)\delta(x),\ f(t) \equiv 0,\ t < 0, \tag{4}$$

Berilgan $b(y), \mu(y), \rho_s(y), f(y)$ funksiyalarga ko‘ra (1)-(4) masala korrekt bo‘lib, ixtiyoriy chekli t larda kompakt tashuvchiga ega bo‘lgan $u(x,y,t)$, $v(x,y,t)$ funksiyalarga nisbatan aniqlaydi.

Masalan, geofizikada sohaning chegarasida muhit nuqtalarining siljish o‘zgarishi bo‘yicha

$$u|_{y=0} = F(x,t), (x,t) \in R_+^2 \tag{5}$$

muhitning strukturasini (bu holda $b(y), \mu(y), \rho_s(y)$ funksiyalar) aniqlash masalasi muhim hisoblanadi.

Bu masala, suyuqlik bilan to‘yintirilgan g‘ovak muhitda SH to‘lqinli tenglamalar sistemasi uchun teskari dinamik masala bo‘ladi. Aksariyat hollarda $f(t)$ funksiya sifatida, Dirakning $\delta(t)$ delta-funksiyasi yoki $t=0$ da chekli uzilishga ega regulyar funksiya tanlanadi.

SH to‘lqinli tenglamalar sistemasi uchun quyidagi teskari masalalar qo‘yiladi:

Masala 1. (5) ma’lumotga ko‘ra (1)-(4) masaladan $\mu(y)$ va $f(t)$ funksiyalarni tiklash (bunda qolgan $\rho_s(y), b(y)$ funksiyalar ma’lum deb hisoblanadi).

Masala 2. (5) ma’lumotga ko‘ra (1)-(4) masaladan $\chi(y)$ va $f(t)$ funksiyalarni tiklash (bunda qolgan $\rho_s(y), \mu(y), \rho_l(y)$ funksiyalar ma’lum deb hisoblanadi).

Masala 3. (5) ma’lumotga ko‘ra (1)-(4) masaladan $\rho_s(y)$ va $f(t)$ funksiyalarni tiklash (bunda qolgan, $\mu(y), b(y)$ funksiyalar ma’lum deb hisoblanadi).

Masala 4. (5) ma’lumotga ko‘ra (1)-(4) masaladan $\rho_l(y)$ va $f(t)$ funksiyalarni tiklash (bunda qolgan $\rho_s(y), \mu(y), \chi(y)$ funksiyalar ma’lum deb hisoblanadi).

Qaralayotgan teskari masalalarda ma’lumlarning ko‘pchiligi (ikki bir o‘zgaruvchili noma’lum bo‘lgan bir paytda berilgan ma’lumotlar ikki o‘zgaruvchili funksiyadan iborat) bu masalani yechish mumkinligi bildiradi. Albatta, bu yerda ham $f(t)$ funksiyaning strukturasi haqida oldindan ma’lum bo‘lgan ba’zi bir aprior farazlarsiz masalani yechib bo‘lmaydi. Haqiqatdan ham, agar $f(t)\equiv 0$ bo‘lsa, u holda $F(x,t)\equiv 0$ bo‘ladi va g‘ovak-elastik dinamik tenglamaga qatnashgan koeffitsiyentlarni topish mumkin emas.

$f(t)$ funksiya quyidagicha

$$f(t)=a\delta(t)+\hat{f}(t)\theta(t), a\neq 0, \qquad (6)$$

ko‘rinishga ega, bunda $\theta(t)$-Xevisayda funksiya, $t\geq 0$ da $\theta(t)=1$, $t<0$ da $\theta(t)=0$ $\hat{f}(t)\in C^1[0,T], T>0$.

Shuningdek $b(y), \mu(y), \rho_s(y)$ funksiyalar R_+^2 yarim fazoning chegarasining yaqinida yetarlicha kichik yupqa qatlam $y\in[0,y_0]$, $y_0>0$ da ma’lum deb faraz qilamiz.

Yuqoridagi farazlarga ko‘ra $(x,t)\in \square\times[0,T], T\succ 0$ nuqtalar to‘plamida $F(x,t)$ funksiyaning berilishiga ko‘ra 1-4 masalalarning biror chekli $[y_0, y_1]$ intervalda yechimini topish imkonini beradi. Shuningdek bu masalalarning yechimi turg‘unligi baholari olindi.

(1)-(4) masalani y koordinata o‘rniga z koordinataga ushbu

$$z=\int_0^y \frac{d\xi}{c_s(\xi)}$$

bunda $c_y=\sqrt{\mu(y)/\rho_s(y)}$ -g‘ovak muhitda ko‘ndalang seysmik to‘lqinning tarqalish tezligi, munosabatlar orqali uning ko‘rinishini o‘zgartiramiz.

Shunday yo'l bilan z koordinataga o'tilgandan so'ng g'ovak muhitda seysmik to'lqin tarqalish tezligi aynan birga teng bo'lib qoladi. Hosil qilingan sistemaga x o'zgaruvchi bo'yicha Fu're almashtirishini qo'llab, (1)-(2) masala

$$\hat{u}_{tt} = \hat{u}_{zz} + \frac{\sigma'}{\sigma}\hat{u}_z - \frac{b(z)\rho_l(z)}{\rho_s(z)}\hat{u}_t + \left[\frac{b^2(z)\rho_l(z)}{\rho_s(z)} - \xi^2 c_s^2(z)\right]\hat{u} + \hat{f}(\xi, z, t) \quad (7)$$

$$\hat{u}|_{t<0} \equiv 0, \quad \hat{u}_z|_{z=0} = f(t) \quad (8)$$

$$\hat{u}|_{z=0} = \hat{F}(\xi, t), \quad (\xi, t) \in R_+^2 \quad (9)$$

bu yerda, $\sigma = \sqrt{\mu\rho_s}$, $f(t) = f(t)/\sigma(0)$,

$$\hat{f}(\xi, z, t) = -\frac{b^3(z)\rho_l(z)}{\rho_s(z)}\int_0^t \hat{u}(\xi, z, \tau)e^{-b(z)(t-\tau)}d\tau$$

ko'rinishga keladi.

(6)-(7) boshlang'ich-chegaraviy masalani yechgandan so'ng $\hat{v}(x, y, t)$ funksiya sohaning bo'lagida

$$\hat{v}(\xi, z, t) = b(z)\int_0^t \hat{u}(\xi, z, \tau)e^{-b(z)(t-\tau)}d\tau$$

formula bilan topiladi.

$G = R_+^2 \times R,\ R_+^2 = \{(x, y) \in R^2 \mid y > 0\}$ yarim fazoda giperbolik tenglamalar sistemasi uchun qo'yilgan (1)-(4) to'g'ri dinamik masalaning yechimining turg'unlik baholarini olamiz.

Quyidagi teorema o'rinli:

Teorema. Aytaylik, $f(t)$ (6) ko'rinishda bo'lib, T>0 da $\hat{f}(t) \in C^1[0, T]$ va $\sigma(z) \in C^1[0, T]$ bo'lsin. U holda ξ parametrining har bir tayinlangan qiymatida $t < z$ da (7),(8) masalaning yechimi aynan nolga teng, $(x, y) \in D(T)$, $D(T) = \{(x, t) \mid 0 \le z \le t \le T - z\}$ nuqtalar uchun $C^2(D(T))$ funksional sinfga tegishli va yechim uchun

$$\left\|\hat{u}\right\|_{C^2(D(T))} \le C\left(|a| + \left\|\hat{f}\right\|_{C^2[0,T]}\right)$$

baho o'rinli, bu yerda C-o'zgarmas, T, ξ $\|\sigma\|_{C^2[0,T/2]}$, $\|\rho_s\|_{C^2[0,T/2]}$, $\|\rho_l\|_{C^2[0,T/2]}$ va $\|\chi\|_{C^1[0,T/2]}$ ga bog'liq. Shu bilan birga

$$w(\xi_1, \xi_2, z, t) = \frac{\partial^2}{\partial t^2}(\hat{u}(\xi_1, z, t) - \hat{u}(\xi_2, z, t))$$

Funksiya ixtiyoriy tayinlangan ξ_1, ξ_2 da $C^2\left(D(T)\right)$ sinf funksiyasi boladi

Isbot (7), (8) munosabatlardan (z,t) $\in D(T)$ uchun

$$\tilde{u}_{tt} - \tilde{u}_{zz} = \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z}\right)(\tilde{u}_t + \tilde{u}_z = \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)(\tilde{u}_t - \tilde{u}_z$$

o‘rinli bo‘lsa, birinchi tartibli differensial operatorlarning mos xarakteristikalari bo‘yicha integrllash orqali

$$\left(\tilde{u}_t + \tilde{u}_z\right)(\xi, z, t) = \int_z^{(z+t)/2} g(\xi, \varsigma, t + z - \varsigma)\, d\varsigma,$$

$$\left(\tilde{u}_t - \tilde{u}_z\right)(\xi, z, t) = -2\hat{f}(t - z) + \int_0^{(t-z)/2} g(\xi, \varsigma, t - z - \varsigma)\, d\varsigma +$$

$$+\int_0^z g(\xi, \varsigma, t - z + \varsigma)\, d\varsigma,$$

tengliklarni olamiz. Bunda

$$g(\xi, z, t) = \left(\frac{\sigma'}{\sigma} u\right)_z - \frac{\chi \rho_l^2}{\rho_s} u_t + q(\xi, z) u + f(\xi, z, t),$$

$$q(\xi, z) = \tilde{q}(\xi, z) - \left(\frac{\sigma'}{\sigma}\right)', \ \tilde{q}(\xi, z) = \frac{b^2 \rho_l}{\rho_s} - \xi^2 c_s^2(z).$$

bundan

$$\tilde{u}(\xi, z, t) = -\hat{f}(t - z) + \frac{1}{2}\int_0^{(t-z)/2} g(\xi, \varsigma, t - z - \varsigma)\, d\varsigma +$$

$$+\frac{1}{2}\int_0^z g(\xi, \varsigma, t - z + \varsigma)\, d\varsigma + \frac{1}{2}\int_0^{(t+z)/2} g(\xi, \varsigma, t + z - \varsigma)\, d\varsigma, \tag{10}$$

$$\tilde{u}_z(\xi, z, t) = \hat{f}(t - z) - \frac{1}{2}\int_0^{(t-z)/2} g(\xi, \varsigma, t - z - \varsigma)\, d\varsigma -$$

$$-\frac{1}{2}\int_0^z g(\xi, \varsigma, t - z + \varsigma)\, d\varsigma + \frac{1}{2}\int_0^{(t+z)/2} g(\xi, \varsigma, t + z - \varsigma)\, d\varsigma, \tag{11}$$

$$\tilde{u}(\xi, z, t) = -\frac{a}{2} - \int_0^{t-z} \hat{f}(\tau)\, d\tau + \frac{1}{2}\int_z^t \int_0^{(\tau-z)/2} g(\xi, \varsigma, \tau - z - \varsigma)\, d\varsigma\, d\tau +$$

$$+\frac{1}{2}\int_z^t \int_0^z g(\xi, \varsigma, \tau - z + \varsigma)\, d\varsigma\, d\tau + \frac{1}{2}\int_z^t \int_z^{(\tau+z)/2} g(\xi, \varsigma, \tau + z - \varsigma)\, d\varsigma\, d\tau. \tag{12}$$

$\tilde{u}_t, \tilde{u}_z$ funksiyalarni t o‘zgaruvchi bo‘yicha hosilalarini hisoblaymiz.

(10), (11) formulalardan

$$\tilde{u}_{tz}(\xi, z, t) = \hat{f}'(t - z) - \frac{1}{4}\Big(g\big(\xi, (t - z)/2, (t - z)/2\big) - g\big(\xi, (t + z)/2, (t + z)/2\big)\Big) -$$

$$-\int_0^{(t-z)/2} g_t(\xi, \varsigma, t - z - \varsigma)\, d\varsigma + \frac{1}{2}\int_0^z g_t(\xi, \varsigma, t - z + \varsigma)\, d\varsigma +$$

$$+\frac{1}{2}\int_{z}^{(t+z)/2} g_t\left(\xi,\varsigma,t+z-\varsigma\right)d\varsigma, \tag{13}$$

$$\tilde{u}_{tt}\left(\xi,z,t\right)=-\hat{f}'(t-z)+\frac{1}{4}\Big(g\left(\xi,(t-z)/2,(t-z)/2\right)+g\left(\xi,(t+z)/2,(t+z)/2\right)\Big)+$$
$$+\int_0^{(t-z)/2} g_t\left(\xi,\varsigma,t-z-\varsigma\right)d\varsigma+\frac{1}{2}\int_0^{z} g_t\left(\xi,\varsigma,t-z+\varsigma\right)d\varsigma+$$
$$+\frac{1}{2}\int_{z}^{(t+z)/2} g_t\left(\xi,\varsigma,t+z-\varsigma\right)d\varsigma. \tag{14}$$

kelib chiqadi.

(10)-(14) sistema $D(T)$ sohada Volterra tipidagi integral tenglama bo‘ladi va ikki marta uzluksiz differensiallanuvchi yagona yechimga ega.

$$w(\xi_1,\xi_2,z,t)=\frac{1}{4}\Big(g\left(\xi_1,(t-z)/2,(t-z)/2\right)+g\left(\xi_1,(t+z)/2,(t+z)/2\right)\Big)+$$
$$+\int_0^{(t-z)/2} g_t\left(\xi_1,\varsigma,t-z-\varsigma\right)d\varsigma+\frac{1}{2}\int_0^{z} g_t\left(\xi_1,\varsigma,t-z+\varsigma\right)d\varsigma+$$
$$+\frac{1}{2}\int_{z}^{(t+z)/2} g_t\left(\xi_1,\varsigma,t+z-\varsigma\right)d\varsigma-$$
$$-\frac{1}{4}\Big(g\left(\xi_2,(t-z)/2,(t-z)/2\right)-g\left(\xi_2,(t+z)/2,(t+z)/2\right)\Big)-$$
$$-\int_0^{(t-z)/2} g_t\left(\xi_2,\varsigma,t-z-\varsigma\right)d\varsigma-\frac{1}{2}\int_0^{z} g_t\left(\xi_2,\varsigma,t-z+\varsigma\right)d\varsigma-$$
$$-\frac{1}{2}\int_{z}^{(t+z)/2} g_t\left(\xi_2,\varsigma,t+z-\varsigma\right)d\varsigma.$$

funksiyani qaraylik.

Bundan $w\in C^1\left(D(T)\right)$ ekanligi ko‘rinib turibdi. bundan tashqari, bu funksiya uchun

$$\|w\|_{C^1(D(T))}\le C\left(|a|+\left\|\hat{f}\right\|_{C^1[0,T]}\right),$$

baho o‘rinli. Bu teorema shartidagi baholashga o‘xshash bo‘lib C o‘zgarmas faqat $T,\xi_1,\xi_2,\|\mu\|_{C^2[0,T/2]},\|\rho_s\|_{C^2[0,T/2]},\|\chi\|_{C^1[0,T/2]}$ va $\|\rho_l\|_{C^1[0,T/2]}$. ga bog‘liq teorema isbotlandi.

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