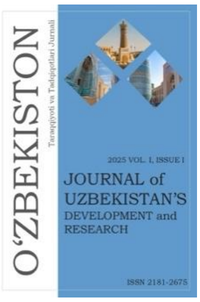

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KESKINLASHISH JARAYONLARINI SONLI MODELLASHTIRISH

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KEYWORDS

keskinlashish, avtomodel yechim, issiqlik tarqalish tenglamasi, modellashtirish.

ABSTRACT

Nochiziqli muhitlarda keskinlashish jarayonlarini ifodalovchi hodisalarni matematik modellashtirish, tadqiq qilish, sonli yechish muhimdir. Shu sababli nochiziqli differensial tenglamalarni yechish uchun olib borilayotgan tadqiqotlar dolzarb va zarur hisoblanadi. Ushbu maqolada shunday jarayonlardan biri issiqlik tarqalish masalasi avtomodel yechimi va sonli yechimi hosil qilingan.

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Quyidagi ko'rinishdagi tenglama boshlang'ich va chegaraviy shartlar bilan berilgan bo'lsin:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(u^{\sigma}\frac{\partial u}{\partial x}\right) + u^{\beta} \qquad (1)$$

$$u\big|_{t=0} = u_0(x) \geq 0, \quad x \in R^N \qquad (2)$$

$$u\big|_{x=0} = u_1(t), \quad u_1(0) \neq 0 \qquad (3)$$

$$u\big|_{x=l(t)} = 0 \qquad (4)$$

Ushbu (1) tenglama issiqlik tarqalish, shuningdek, turli fizik jarayonlarni ifodalaydi. Yuqoridagi masala va uning yechimi yuzasidan ko'plab olimlar izlanish olib brogan. Shular jumlasidan A.A. Samarskiy, V.A. Galaktionov, S.P. Kurdyumov, A.P. Mixaylov, J. Vazquezlarni misol sifatida keltirish mumkin. Ular o'z ilmiy izlanishlarida yuqoridagi tenglama yechimlari ustida bosh qotirganlar. Yurtimiz olimlaridan M.M. Aripov, J.O'. Muhammadiyev, A.T. Xaydarov, A.S. Matyakubov va ularning shogirdlari ushbu masala yuzasidan o'z ilmiy ishlarini olib borganlar.

Dastlab (1) tenglama uchun avtomodel yechim quramiz:

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$$u_A(t,x) = (T_0 - t)^{-\frac{1}{\beta-1}}\eta_A(\xi), \qquad \xi = \frac{x}{(T_0 - t)^n} \qquad (5)$$

bunda

$$n = \frac{\beta - (\sigma+1)}{2(\beta-1)}$$

bu yerda, $T_0 > 0$ - tenglamaning yechimi mavjud bo'ladigan vaqt. Agar $t \geq T_0$ bo'lsa, yechim umumiy holatda aniqlanmagan bo'ladi. $t \to T_0^-$ da yechim amplitudasi cheksiz o'sib boradi.

(5) belgilashlarni (1) tenglamaga qo'yish orqali quyidagicha ifoda hosil qilamiz:

$$\nabla_\xi(\eta_A^0 \nabla_\xi \eta_A) - n\nabla_\xi \eta_A \xi - \frac{1}{\beta-1}\eta_A^0 + \eta_A^\beta = 0 \qquad \xi \in R^N \qquad (6)$$

(2.4) tenglama $\eta_A \equiv 0$ trival yechimi mavjud. Shuningdek,

$$\eta_A(\xi) \in \eta_H = (\beta-1)^{-\frac{1}{\beta-1}} \qquad (7)$$

ko'rinishdagi gomotetrik yechimi ham bor. (1) tenglamaga ko'ra ushbu yechim bir jinsli fazoda mavjud bo'ladi.

Quyidagi radial simmetrik yechimlarni ko'rib chiqamiz:

$$\xi = \frac{p}{(T_0 - t)^n}, \qquad p = |x| \qquad (8)$$

Shunda (6) tenglama oddiy differensial tenglama ko'rinishiga keladi:

$$\frac{1}{\xi^{N-1}}(\xi^{N-1}\eta_A^0\eta_A')' - n\eta_A'\xi - \frac{1}{\beta-1}\eta_A + \eta_A^\beta = 0, \qquad \xi > 0 \qquad (9)$$

Birinchi operatorni quyidagi ko'rinishda yozish mumkin:

$$(\eta_A^0\eta_A')' + \frac{N-1}{\xi}\eta_A^0\eta_A'$$

Shuning uchun yechim η_A butun R^N fazoda aniq bo'lishi uchun quyidagi simmetriya shartlari bajarilishi zarur:

$$\eta_A'(0) = 0, \qquad \eta_A(0) > 0 \qquad (10)$$

Bundan tashqari, quyidagi shart ham bajarilishi kerak:

$$\eta_A(\infty) = 0 \qquad (11)$$

S-rejimda lokalizatsiya (β = σ + 1). Bu holda (5) tenglama eng oddiy ko'rinishga keladi:

$$\frac{1}{\sigma+1}\Delta_\xi \eta_A^{\sigma+1} - \frac{1}{\sigma}\eta_A + \eta_A^{\sigma+1} = 0, \qquad \xi \in R^N \qquad (12)$$

Shu bilan birga, radial simmetrik masala (8)-(10) quyidagicha yoziladi:

$$\frac{1}{\xi^{N-1}}(\xi^{N-1}\eta_A^\sigma\eta_A')' - \frac{1}{\sigma}\eta_A + \eta_A^{\sigma+1} = 0 \qquad \xi > 0 \qquad (13)$$

$\eta'_A(0)=0 \quad (\eta_A(0)>0) \qquad \eta_A(\infty)=0$ (14)

N=1 bo'lgan holat uchun. Bir o'lchamli bo'lganda (13) tenglama avtonom tenglamaga aylanadi va integrallanadi. Xususan, quyidagi ko'rinishdagi yechim hosil bo'ladi:

$$\eta_A(\xi)=\left(\frac{2(\sigma+1)}{\sigma(\sigma+2)}\cos^2\frac{\pi\xi}{L_S}\right)^{\frac{1}{\sigma}} \qquad \xi\geq 0 \qquad (15)$$

bu yerda, $L_S=\frac{2\pi}{\sigma}\sqrt{\sigma+1}$

Agar $\beta=\sigma+1$ bo'lsa, (1) tenglamadan ko'rinadiki, $\xi=x$ ya'ni, (1) tenglamaning yechimini o'zgaruvchilarga ajraluvchi yechim shaklida yozish mumkin:

$$u_A(t,x)=(T_0-t)^{-\frac{1}{\sigma}}\eta_A(x) \qquad 0<t<T_0, \quad x\in R \qquad (16)$$

η_A funksiyasi juft funksiya, ya'ni u x ning manfiy qiymatlarida ham simmetrik tarzda uzaytiriladi.

(16) yechim issiqlikning diffuzion muhitlardagi tarqalishi holatidir. Gap shundaki, (16) dagi $\eta_A(x)$ funksiya davriy hisobalanadi, ushbu funksiya quyidagi nuqtalarda nolga teng bo'ladi:

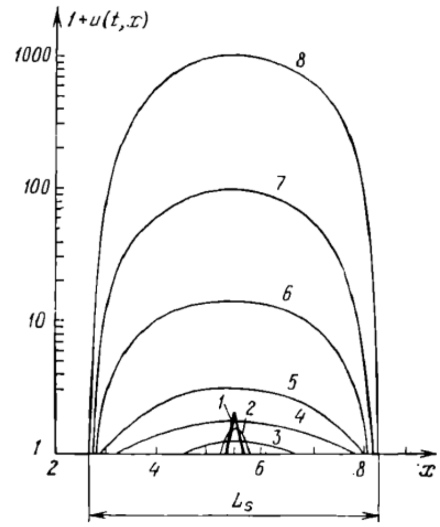
$$x_t=\left(\frac{1}{2}\pm k\right)L_S \quad (k=0,1,2,\ldots.) \qquad (17)$$

shuningdek, $x\to x_k$ da $\eta_A^{\sigma}\eta'_A\to 0$ va uzluksiz bo'ladi.

Avtomodel yechim quyidagi funksiyadan iborat bo'ladi:

$$u_A(t,x)=\begin{cases}(T_0-t)^{-\frac{1}{\sigma}}\left(\frac{2(\sigma+1)}{\sigma(\sigma+2)}\cos^2\frac{\pi\xi}{L_S}\right)^{\frac{1}{\sigma}}, & |x|<\frac{L_S}{2}\\ 0, \quad |x|\geq\frac{L_S}{2} & \end{cases} \qquad 0<t<T \qquad (18)$$

Ushbu (18) tenglama S rejimdagi avtomodel yechim hisoblanadi. Bu struktura lokalizatsiyalangan, ya'ni u faqat $|x|<\frac{L_S}{2}$ da mavjud bo'ladi va bu vaqt oralig'ida mavjudlikni saqlab qoladi. Yechimning grafikda ko'rinishi 1-rasmdagi kabi bo'ladi:

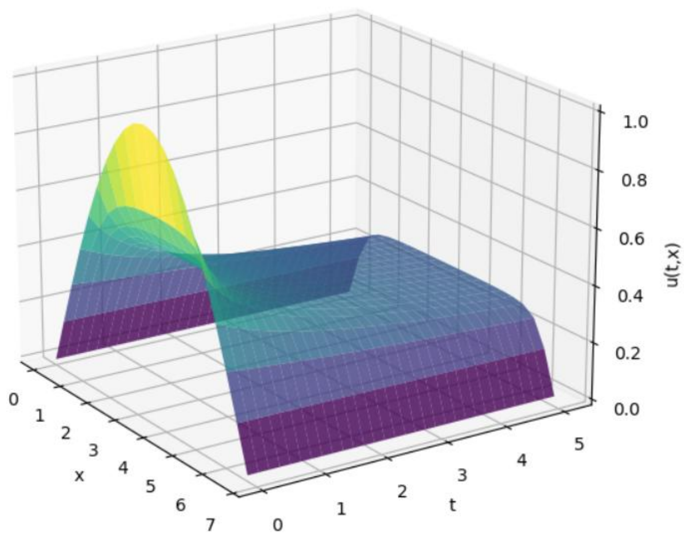


(1-rasm)

Ushbu rasm parametrlarning $\sigma = 2$, $\beta = 3$, $N = 1$ bo'lganda qay holatda ko'rinishi ko'rsatib o'tilgan.

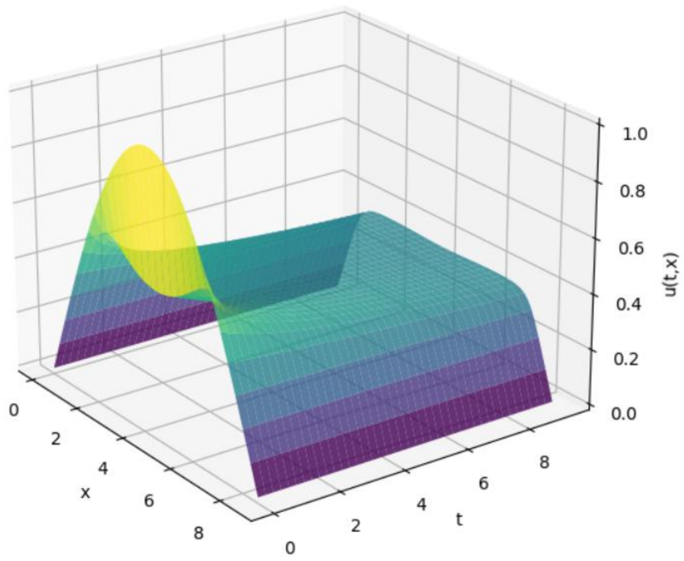
Quyida (1) tenglama uchun Python dasturlash tilida tayyorlangan dasturning tenglama parametrlari turli qiymatlaridagi 3 o'lchamli fazoda grafigi ko'rsatib o'tiladi:

1. Parametrlarning $\sigma = 6.5,\ \beta = 4.2,\ T = 5.1$ qiymatlaridagi grafik:



(2-rasm)

2. Parametrlarning $\sigma = 8.1,\ \beta = 6.7,\ T = 9.3$ qiymatlaridagi grafik:



(3 -rasm)

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